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MATHEMATICS/ FURTHER MATHEMATICS

REPORT ON COMPONENTS
TAKEN IN JUNE 2000

Subject 9200

Paper 1

General Comments

The paper this year appeared to be more accessible than the corresponding paper set in 1999. The range of questions and length of the paper allowed candidates to demonstrate their knowledge and skills without great pressure due to time or particularly difficult questions. However, to achieve a very high score candidates not only had to display a sound and thorough understanding of the syllabus, but show attention to detail often lacking in their less successful peers.

Most candidates worked through the questions in numerical order, perhaps returning to questions at the end of the paper. In general, candidates who adopt this approach are more successful than those who answer the questions in order of preference. No question was consistently omitted by a large number of candidates, although there was considerable evidence of strengths and weaknesses in certain topic areas. In general, candidates did not waste time in fruitless work, although techniques adopted by some in Q9, Q12 and parts of Q16 did result in lost time.

A particular concern this year was a lack of appreciation of the requirements of exact working (Q5, Q8, Q12, Q13). Answers requested in this form were very often found using a calculator and given as approximations; a significant number of candidates confuse an **exact** answer resulting from an **exact** method with a **very accurate** answer resulting from a **non-exact** method.

General standards of accuracy, both in numerical and algebraic work, were fair, although a number of Examiners commented on the high levels of carelessness exhibited by some candidates. Confusion between degrees and radians, and some weaknesses in algebraic manipulation were also evident. Presentation was generally good, although the practice of splitting the page into two columns should be strongly discouraged. The paper contained a number of questions where the answer was given (Q10, Q12, Q13, Q14, Q15, Q16), and candidates need to develop further their skills in providing sufficient detail for what may seem, to them, to be an obvious solution. In situations where some explanation is required (Q4, Q6, Q14, Q16), candidates need practice in providing detailed but succinct answers.

Comments on Individual Questions

Q1 For the majority of candidates this proved a successful start to the paper. The most common errors seen were (0, -1) for the centre of C and $\sqrt{24}$ for the radius. It was unfortunate for a large number of candidates, however, that they seemed unfamiliar with the $(x-a)^2+(y-b)^2=r^2$ form of the equation of a circle, and a great deal of time and effort was spent rearranging the given equation to the form $x^2+y^2+2fx+2gy+c=0$, with errors quite frequently appearing in this method

radius = 5 centre (0, 1)

Q2 For most candidates, this was another successful question, with the vast majority obtaining the equation of the straight line without difficulty. The second part of the question proved more challenging; a number of candidates successfully substituted into their straight line equation, but failed to obtain t correctly, and Examiners were disappointed to note a number of candidates failing to clear the denominator. Other candidates did not appreciate the link with the first part; these generally attempted either to substitute one of the two given equations into the other, or to find $\frac{dy}{dx}$ through an adaptation of parametric differentiation.

$y=3x-1$ $\qquad t=3s^2-s^3$.

Q3 Although successfully answered by a large number of candidates, there was a wide variety of errors, especially in (i). In each part Examiners were looking for numerical evidence (stated or implied) of the intercepts. The use of graph paper was not expected, although a number of candidates used up crucial time in this way.

(i) Many candidates used scale factor ½ parallel to both axes, resulting in intercepts of (0, 1½) and (½, 0). Other common errors included intercepts (0, 6) with (½, 0) or (1, 0), (0, 3½) with (½, 0) or (1, 0) and (0, 3) with (½, 0).

(ii) Generally well answered, reflection in the y-axis being the most common cause of error.

(iii) Again, generally well answered, although some curves resulting from reflections in $y = x$ had discontinuities in their gradients on crossing this line, or did not meet both axes.

Q4 Another successful question for a large number of candidates. Errors in (i) were very rare. In (ii), quite a number of candidates confused u_n with S_n (the sum of the first n terms). Some candidates were clearly unfamiliar with the notation used in the question and failed to answer this part. For those who made a sound attempt, most common errors included $a = 1$ or 7, and $d = 3$ or 7. Candidates should be discouraged from giving their answers in the form '$a = 3$, $d = 4$ where $u_n = a + (n-1)d$'. In (iii), most candidates correctly identified v_n as a convergent sequence, but many of the reasons given were not sufficiently accurate. For example, 'v_n gets smaller and smaller' or 'u_n gets larger and larger' do not necessarily imply convergence. Reference to the divergence of u_n, either directly or as 'u_n tends to infinity' or the limit of v_n being zero were necessary to achieve the mark for this part.

(i) 7, 11, 15, (ii) $3 + 4(n-1)$.

Q5 Although a significant number of good solutions were seen, it was evident that some candidates were unfamiliar with the modulus notation used in this question. The majority of candidates attempted the first part by squaring, although $(y-1)^2 = 6$ was common, as were errors in solving the ensuing quadratic correctly. Failure to use the solutions to the quadratic to form an inequality was seen, as was confusion between y and x. Few opted for the more simple $-6 < y - 1 < 6$ leading directly to $-5 < y < 7$.

For the second part of the question, a significant number of candidates failed to recognise the link to the first part, often leading to errors such as $x\ln 2 - \ln 1 < \ln 6$, or a quadratic in 2^x which led to difficulties when taking logarithms. Many candidates were successful in finding $x < \frac{\ln 7}{\ln 2}$ and clearly had a sound understanding of how to deal with x as an index in an equation, although a significant number of these included the logarithm of -5 in some form, losing one mark. Other errors included expressing the answer in terms of log rather than ln.

$-5 < y < 7$, $x < \frac{\ln 7}{\ln 2}$.

Q6 A well answered question for many candidates. In the first part, Examiners required numerical evidence (typically $2 - \tan 1 = 0.4$ and $2.4 - \tan 1.2 = -0.2$) **and** a clear explanation, including identification of a change of sign and appropriate conclusion concerning the root. Some numerical errors were seen, as was confusion between degrees and radians.

Most candidates displayed sound knowledge of the Newton-Raphson method, although many wasted valuable time calculating third, fourth and fifth approximations. The differentiation of $2x - \tan x$ was generally good, the most common error resulting in $x - \sec^2 x$. Unfortunately, a significant number made errors in finding $2 - \sec^2(1.1)$, often arising when $2 - \sec^2 x$ became $1 + \tan^2 x$ or from errors in using the calculator. As a result, many well-intentioned candidates failed to obtain the correct second approximation. Candidates should be encouraged to show their intentions in calculations such as these, as an incorrect numerical answer with no evidence of a method is worthless.

1.18.

Q7 The vast majority of candidates were able to make a good attempt at this question, but few achieved full marks. Techniques for finding partial fractions are clearly well known, although examiners were concerned that a number of candidates did not display a sound grasp of the underlying mathematics, relying rather too heavily on 'learnt' strategies. In particular, some who attempted the 'cover-up' method and a few others failed to substitute the correct coefficient into the correct fraction.

Most candidates were able to obtain $\ln x$ and $\ln(x-2)$. There was, however, confusion over where the coefficients found in the first part should appear. It was common to see 2 in the numerator, and omission of the ½ altogether was a particular problem for those who chose to work with $\ln(2x)$ and $\ln(2x-4)$. Of the candidates who successfully and accurately integrated, only very few achieved a final mark by using modulus signs in both parts of the answer **and** an arbitrary constant.

$\frac{1}{2(x-2)} - \frac{1}{2x}$ $\frac{1}{2}\ln|x-2| - \frac{1}{2}\ln|x| + c$.

Q8 Although some candidates were clearly very well prepared for the non-calculator approach required by this question, it was disappointing that a large number of candidates were successful only in (i) and (iii). In (i), the sine rule was well applied, a very few failing to find a value for $\sin\theta$. Unfortunately, most candidates approached (ii) using tables or the calculator, and were unable to provide an exact value for $\cos\theta$. Successful candidates generally used a Pythagorean argument based on the right-angled triangle with sides 2, $\sqrt{21}$ and 5, with surprisingly few opting to use the identity $\sin^2\theta + \cos^2\theta = 1$. Most candidates successfully found cosBDC and sinBDC, although a large number lost time in lengthy working, failing to spot the 3, 4, 5 triangle. A very large number of candidates seemed unfamiliar with the identity to expand sin(BDC + BDA), many asserting that sinADC = sin ADB + sinBDC, leading to sinADC = $\frac{6}{5}$. Of those who attempted the correct expansion, most were successful in obtaining the exact value required.

(i) $\frac{2}{5}$, (ii) $\frac{\sqrt{21}}{5}$, (iii) $\frac{3}{5}, \frac{4}{5}$, (iv) $\frac{6+4\sqrt{21}}{25}$.

Q9 Many candidates wasted valuable time here, spending a great deal of time expanding the given expressions but often failing to reach any solution at all. In (i), most candidates successfully found A = 4, but relatively few of these were able to write this down directly through a consideration of coefficients of x^3. Some used the Factor Theorem successfully to find $(x + 5)$ as a factor, but either failed to deduce B = 5 or made little further progress. Expansions of the given expression often revealed a lack of understanding of either the Binomial Theorem or a simpler technique using Pascal's Triangle, and those who simply multiplied out often made arithmetical errors. In some cases, the binomial coefficients were lost altogether, in others A appeared only in the first term, and there were many who failed to include the Cx term when equating coefficients of x or asserted that C = 299 and BC = 495.

In (ii), a large number overlooked more sophisticated solutions in favour of multiplying out and collecting terms, often using the factor theorem to deduce that $y = 2$ was a solution. Subsequent long division of the (expanded) cubic to leave a quadratic often led to a full correct solution, but many failed to proceed beyond this point. Of those who spotted that $(y-2)$ was a factor, many simply divided through to leave a quadratic, so losing a solution. The neat substitution $z = y - 2$ was often spoiled by a failure either to find values of y for each z, or to obtain both positive and negative roots of the quadratic. Candidates should be advised that solutions of the form $2 \pm \frac{2}{3}$ are not accepted. A few unfortunate candidates successfully factorised the expanded cubic but failed actually to solve.

(a) A = 4, B = 5, C = –1; (b) $y = 2, \frac{4}{3}, \frac{8}{3}$.

Q10 A generally well answered question, although some candidates lost marks through a failure to read the question thoroughly. In (i), a correct method (usually using the scalar product, more rarely the cosine rule using lengths AO, BO and AB) was generally applied, but the request for 3 decimal places was sometimes ignored. Although (ii) was often correct, a significant minority misunderstood the situation, repeating the coordinates of A and B or obtaining (–3, 4, –0.1) and (1, 2, –0.5). With the correct coordinates for A′ and B′, most found A′OB′ correctly, although some failed to preserve sufficient accuracy for use in the relative error. (iii) was generally sound, although a common error was to use AOB instead of A′OB′ in the denominator. As is often the case when answers are given, Examiners were disappointed to see some candidates attempting to 'fudge' the answer.

(i) 63.851°, (ii) (–3, 4, 0), (1, 2, 0).

Q11 It was pleasing to note that a significant number of candidates made successful attempts at this question. However, there were many others who struggled with several aspects of the question. In (i), the technique for finding R and α was well understood, although $\alpha = 16°$ arising from $\tan\alpha = \frac{7}{24}$ was not uncommon (a follow through mark was available in this case). Unfortunately, a wide variety of errors were seen in the solution of $\sin(\theta + 74°) = -\frac{3}{25}$; confusion with (or complete omission of) the minus sign was common, and surprisingly few were able confidently to find more than one correct value for $\sin(\theta + 74°)$, some also including spurious extra values (173.1° leading to $\theta = 99°$ was common). Examiners penalised candidates for extra solutions in the given range, although the common presence of solutions outside $-90° < \theta < 270°$ was overlooked.

In (ii), many candidates failed to spot the link with the first part of the question, often opting to restart. Of these, quite a number were successful, although errors were seen using $\sin(y - \alpha)$ in (b), as were solutions involving $\cos(y - \alpha)$ but still using –6.9°, and a surprising number obtained $x - 10° = -6.9°$ etc., overlooking the need to incorporate α at this stage. In (a), many did choose to simply add 10° to the previously found solutions (follow through marks available here and in (b) for answers in the correct range, with extra solutions now overlooked), although some expressed this intention but then subtracted! Some thought (b) identical to the equation in (i), while others adopted the dangerous technique of using arcsin followed by cos and arccos followed by sin on their solutions to (i). Only a very few used the relationship $y = 90 - \theta$.

(i) $R = 25$, $\alpha = 74°$, $\theta = -81°, 113°$, (ii)(a) 123°; (b) 171°.

Q12 Although some excellent solutions were seen, a large number of candidates encountered substantial difficulties with many of the key principles here. In the first part, most attempted to find $dx = \cos\theta\, d\theta$, but $d\theta = \cos\theta\, dx$ was quite common, as was a failure to substitute correctly (or at all) for dx in the given integral. Knowledge of the basic identity was generally sound, but of those who reached $\sin^2\theta\cos^2\theta$, many attempted to 'fudge' to the given answer. Examiners were looking for a clear indication that a double angle formula had been used, at least through seeing $\frac{1}{4}\int 4\sin^2\theta\cos^2\theta\, d\theta$; candidates must be encouraged to show sufficient working whenever questions such as these arise.

For the area calculation, a substantial number, clearly with no known method to adopt, made fundamental errors in integrating (answers such as $\sin^3 2\theta/\cos 2\theta$ were not uncommon). For those who attempted a double angle approach, many made errors in the sign or omitted the ½, or arrived at $a + b\cos 2\theta$, and the subsequent integration was often incorrect; the omission of $\frac{1}{4}$ in the integration of $\cos 4\theta$ was common, as were sign errors. Some candidates, having successfully integrated, failed to provide an exact answer.

The volume caused further difficulties, with many attempting an integration involving θ based around the previously given area integral ($\int \sin^4 2\theta \, d\theta$ being common). A significant number did progress to $\pi \int x^4(1-x^2)\mathrm{d}x$, but either integrated incorrectly (sometimes caused by errors in multiplying out the above integral) or failed to provide an exact answer. The use of integration by parts was uncommon but occasionally successful.

$$A = \frac{1}{8}\left(\frac{\pi}{6} - \frac{\sqrt{3}}{8}\right), \quad V = \frac{23\pi}{4480}.$$

Q13 This was a successful question for a large proportion of the candidates, the only significant problems arising from arithmetical errors and failure to use exact methods. In (i), the equation of l' was not always in the form requested, and there were some errors in finding the correct gradient from the given line; these resulted either from a poor (or entirely absent) attempt at rearranging the equation for l, or from muddled use of -1 in the product of the gradients. (ii) was generally well attempted, although some chose less than efficient strategies, equating $2y - x + 7$ to $2x + y - 4$ to create a third equation. Arithmetical errors were also common. In (iii), the vast majority appreciated the need to work with the two points $(-1, 6)$ and $(3, -2)$, although some weaker candidates were unsure of the method for finding the distance between two points. Others omitted this part altogether. The formula for the perpendicular distance of a point from a line was rare but generally well used if attempted. In (iv), a failure to adopt a strategy which could lead to an exact answer was common, often involving the cosine rule followed by ½$ab\sin C$ for the area or the formula $\sqrt{s(s-a)(s-b)(s-c)}$. A surprising number of candidates found the midpoint of QR, assuming this to be the foot of the perpendicular from P, while others assumed angle QPR to be 90°. Two other exact techniques ($A = 16 \times 13 - 3 \times 13 - 5 \times 5 - 8 \times 8$ and $A = ½ ((-1 \times -7) + (-7 \times 1) + (9 \times 6) - (6 \times -7) - (-7 \times 9) - (1 \times -1))$) were rare but generally well attempted.

(i) $2x + y - 4 = 0$, (ii) $(3, -2)$, (iv) 80.

Q14 This was not, for most candidates, a successful question overall, although a substantial number were able to begin well and achieve 4 or even 6 marks out of 11. The vast majority were able to attempt to establish the differential equation, with the most common error being $\frac{\mathrm{d}m}{\mathrm{d}t} = km$. Those whose differential equation did not require the separation of variables were unable to score further in this section, but for the rest, the process of separation and integration was generally successful, resulting in $\ln m = -kt + c$. Unfortunately, perhaps spurred on by the given answer, many failed to convince Examiners that the subsequent exponentiation was fully understood, with $m = e^{-kt} + e^c$ common, often followed by a 'fudge' leading to the answer. The process of finding the constant was also often not clearly explained, and candidates in the future must be encouraged to show adequate working in questions such as this.

The remaining 5 marks were only rarely obtained. Candidates' graphs often resembled the given graph, featuring negative gradients (resulting in reaching a limit from above rather than below) or a starting value for M which was less than 0. The required limit in (ii) was rare, with m_0 the most common error, often as a result of an incorrect graph. Few candidates had returned to $M = A - m$ in search of the limit. For the final part, most candidates began their answer by stating $\frac{dM}{dt} = km$ and hence $k(A - M)$ (which is evident directly from the answer given), omitting the all important step stating (or deriving from $M = A - m_0e^{-kt}$) that $\frac{dM}{dt} = -\frac{dm}{dt}$, leading to the answer given. Candidates whose writing does not allow M and m to be easily distinguished need to be particularly careful on questions such as this.

$\frac{dM}{dt} = -km$, (ii) $M \to A$.

Q15 For many candidates this was a welcome opportunity at this stage of the paper to score at least 6 marks, although fully correct answers to part (iii) were not common. Most successfully found $A = 14\theta$, although a few candidates either left their answer as $32\theta - 18\theta$ or used an incorrect formula for sector area (either omitting the ½ or introducing $\sin\theta$). In (i), Examiners were not always presented with a completely convincing argument, many rejecting $\frac{dA}{d\theta} \times \frac{d\theta}{dt}$ in favour of a correct argument based on the increase in 1 second. Those who simply moved from 14θ to $14 \times 0.1 = 1.4$ were given the benefit of the doubt; had the question involved an expression which was non-linear, some candidates may have experienced difficulties with their chosen methods. In part (ii), a relatively large number of errors were made in finding the perimeter, with $8\theta - 6\theta$ often appearing, with or without the lengths of MN and PQ. (iii) was a straightforward and almost exclusively successful cosine rule application, although a small number of candidates did omit this part altogether.

The last part of the question presented candidates with a much greater challenge, and there was a wide variety of errors seen. The most worrying were those where $L^2 = 100 - 96\cos\theta$ led to $L = 10 - \sqrt{96\cos\theta}$, but other errors included incorrect implicit differentiation (eg $2\frac{dL}{dt}$ or $L\frac{dL}{dt}$ rather than $2L\frac{dL}{dt}$) and over simplification of the situation, with $\frac{dL}{d\theta}$ treated as $\frac{dL^2}{d\theta}$ to give simply $96\sin\theta$. Sign errors in differentiating $100 - 96\cos\theta$ were also common, as were problems in manipulating the ½ and square root for those who found $\frac{dL}{d\theta} = \frac{1}{2}(100 - 96\cos\theta)^{\frac{1}{2}}$. Some unfortunate candidates successfully found $\frac{dL}{d\theta}$, substituted correctly for θ, but failed ever to involve $\frac{d\theta}{dt}$. The final numerical answer was rare, although a significant minority of candidates produced neat and accurate solutions which showed a sound understanding of the techniques involved.

14θ (i) 1.4, (ii) 1.4, 0.58.

Q16 As might be expected in Q16 of a 3 hour paper, there was some evidence of candidates running a little short of time; however, most were able to attempt the most straightforward parts of the question, and it was not uncommon for even the weakest to pick up 3 or 4 marks.

In (i)(a), most were able to write down the equation in p and q. Examiners allowed the mark for differentiating y even for those who had chosen specific values for p and q (which may or may not have satisfied $4p + 16q = 1$), but no further marks were available in this case. Successful candidates followed the substitution of $\sqrt{2}$ with its extraction as a factor, leaving either $4p + 16q$, $2p + 8q$ or $p + 4q$ to be substituted, or eliminated p or q using the initial equation and simplified to reach the given answer. The value of a in (i)(b) was generally correctly found (arithmetical errors were, however, common), but attempts to expand y were generally less successful. Of those who used the binomial expansion, many made errors with the sign, while a large number of candidates attempted a Maclaurin's expansion, often expending a great deal of time and effort for little or no reward. Candidates should be encouraged in such questions, where possible, to use the simplest method available, and to give their final answer with the value of a substituted in. (i)(c) saw many correct values of k, although a failure to give 3 significant figures was fairly common. Most candidates attempted valid expansions of e^{-x} and e^{x}, although some failed to deal with the constant term or made one or several sign or algebraic errors (typically giving $\frac{-x^2}{2}$ rather than $\frac{(-x)^2}{2}$). Again, a surprising number chose to attempt a Maclaurin's expansion, sometimes with reasonable success, although the constant term was typically badly dealt with here.

In (ii), most candidates failed to provide an adequate explanation for two marks. Examiners were, however, only looking for an indication that the 40 was 20×2 for the first mark. The most successful approaches to justifying the series either saw individual terms (with 0.1, 0.2 etc.) written out followed by the given summations, or resulted from the assertion that the summation was equivalent to the summation of e^x where $x=0.1r$. For the last part of (ii), a significant number of candidates attempted the summation of an arithmetic progression. Of those who correctly identified a geometric progression, common ratios of 0.1 and −0.1 were common (often with incorrect first terms), as was over approximation (sometimes of $e^{0.1}$ and $e^{-0.1}$) leading to an inaccurate answer. Mistakes in the second summation were more common than in the first. A number of candidates had clearly simply added together all the terms on their calculators. While a correct answer obtained from this method will attract full marks, candidates should be aware that in a question such as this, one small slip will cost 3 marks, and should be encouraged to develop the appropriate techniques for use in such questions.

(i)(a) $4p + 16q = 1$; (b) $a = \frac{5}{36}, y = \frac{5}{36}x^2 + \frac{5}{324}x^4$;

(c) $k = 0.181, y = kx^2 + \frac{kx^4}{12}$, (ii) 6.4

Paper 2

General comments

On the whole the paper appears to have been straightforward for many candidates and scores of over 100 out of the maximum 120 marks were quite common. The numbers of candidates, who had mastered little of the syllabus and could only gain extremely low marks, seemed less than in previous years. There seemed to be sufficient time for well-prepared candidates to make reasonable attempts at all the questions and no question seemed to present insuperable difficulties, though question 8, 10 and 16 proved to be quite discriminating.

The work was generally well presented and examiners made few deductions for the omission of essential working. Attention to the accuracy of answers was usually adequate but some candidates continue to over approximate early on in the question and fail to achieve the accuracy required later. This led to the loss of marks in questions 7, 9, 13, 14 and 15 for example.

The tendency of some candidates to write lengthier solutions than necessary has been commented on in previous reports. This practice must use up valuable time, which might be spent on other questions, or on the vital matters of thinking about the strategy and checking the steps of the solution having attempted. In this paper unnecessarily long solutions to questions 2, 6, 8, 9, 10, and 15 were quite frequent.

The detailed comments on individual questions that follow inevitably refer to mistakes and misconceptions and can lead to a cumulative impression of poor work on a difficult paper. It should be emphasised that there were many scripts that showed good and sometimes excellent knowledge and capability over the entire syllabus. Where numerical and other answers are given after the comments on individual questions it should be understood that the alternative forms are often possible and that the form given not the only 'correct' answer.

Comments on individual questions

Q1 Most candidates applied the quotient rule. Some used the product rule and a minority simplified the problem by first dividing. The methods of differentiation were usually applied correctly, but minor errors, often of sign, were common in the ensuing simplification.

$\frac{5}{(2x+1)^2}$.

Q2 This question was generally well answered although it was surprisingly omitted by a number of otherwise well-prepared candidates. Solutions varied considerably in length. Some economical approaches used only the numerical value of one relevant cosine and took half a dozen lines while others evaluated four angles in the course of two pages of work.

4.9.

Q3 The first two parts were done very well but there were not many fully correct solutions to the final part. Most candidates showed some understanding of conditional probability but many failed to apply it correctly in the given context. The misinterpretation P ('No'|'Truthfully') was quite common.

(ii) $\frac{1}{20}$, $\frac{38}{39}$.

Q4 Part (a) was poorly answered. Candidates did not show much knowledge of appropriate diagrams for the different kinds of data being considered and some resorted to giving the same answer in all three parts presumably in the hope that at least one was correct.

In part (b) sounder understanding of the concept of interquartile range was shown than in last years paper. The illustrations in part (ii) varied considerably, from entirely satisfactory box-and-whisker plots complete with a contextual linear scale running from 0 to 60 on the one hand, to rough sketches in which the positions of the five key features bore little or no discernible relation to any imaginable linear scale, on the other.

(a)(i) Pie chart, (ii) Histogram, (iii) Double bar chart; (b)(i) 38, 11.

Q5 This was usually well answered. Most candidates formed and solved a quadratic equation in t. Those who worked with the velocities at a height of six metres, commonly calculated only the time of ascent and made no use of the velocities of descent to find a second time.

0.47s, 2.6s.

Q6 Part (i) was usually answered well and the quickest calculations of the total distance travelled were those which equated it to the area under the graph rather than to the sum of the three distances found using constant acceleration formulae.

Part (ii) discriminated well, rewarding candidates who had a clear idea about the new situation.

(i) 450m.

Q7 The principle of conservation of linear momentum was generally known and applied correctly, though some candidates took the trucks to be moving initially in opposite directions rather than in the same direction, as stated in the question. Answers to the remainder of the question were less successful. Examiners felt that some errors could be avoided if candidates had prepared diagrams showing the forces acting on the separate trucks. In otherwise correct work the final accuracy mark for part (ii) was lost if the rounded value of the acceleration in part (i) was used.

Q8 This question was not done well. Reasonable progress was only made by candidates who showed mechanical and geometrical insight, the remainder tending to become entangled in resolving equations formed using different tensions in the two parts of the string and unrelated angles of inclination. Correct solutions to the second part were rare.

Q9 Examiners were pleased with the work on this question. Most candidates were able to prove the general range equation. Those who started part (i) by substituting in the range equation to obtain two equations in U and D usually did well, but some used up valuable time repeating the bookwork and deriving these equations from first principles. This was another question were some candidates lost accuracy marks by making premature approximations.

Q10 Part (a) was generally done well. This standard situation was clearly familiar and the appropriate equations were set up. By contrast part (b) was done badly. Most candidates could score a mark or two by forming and equation or inequality involving one of the particles but a proper appreciation and precise grasp of the mechanical situation seemed to be beyond all but the best of the candidates. Common errors included omission of the tension in the string and the idea that the frictional forces on the particles were equal in magnitude. Candidates who succeeded in finding the exact critical value of μ were required to justify the direction of the inequality in the given answer and this was not always done satisfactorily.

Q11 Most candidates showed some understanding of sampling but their answers varied considerably in both the precision of expression and the relevance to the particular situation under discussion. However there were some sound and thoughtful responses to this question.

Procedure A is preferable.

Q12 This question was well answered. A fairly common failing was to omit the calculations of E (x) after finding the values of P. Some candidates seemed unclear about the distinction between a 'value' and an 'expression' for, having obtained $\sum x^2 p$ they used the variance formula and concluded prematurely that E $(x) = \pm \sqrt{(6p - \frac{1}{2})}$.

1, $\frac{1}{2}$.

Q13 This too was done well. Many candidates seemed prepared for both parts of this question. In part (i) examiners felt that final answers of 0.05 indicated lack of understanding of the phrase "correct to two significant figures" in the rubric. In part (ii) most candidates were aware of the need for a continuity correction

(i) 0.051, (ii) 0.064.

Q14 This apparently straightforward question was not often answered completely correctly. In part (i) most candidates quoted a correct formula for the unbiased estimate of a variance but failure to substitute numerical values correctly were surprisingly common. In part (ii) the procedure for calculating a confidence interval were clearly well known and only rarely was 1.96 not used as the relevant z-value. The final part proved hazardous. Even though some candidates made potentially helpful sketches of a normal curve, the standardised value of a was often equated to 1.282 rather than −1.282, and following on from the work on the confidence interval, some candidates used the standard error rather than the standard deviation in this part.

(i) 8900, 503 000, (ii) $8740 < \mu < 9060$; 7990.

Q15 Though there were some errors in substituting appropriate values in the formulae for the product moment correlation coefficient and for the gradient of the regression line, candidates were better at calculating values than commenting on their results. Having found the gradient of the regression line, a common error was to complete the work by requiring that the line passed through a data point, usually (20,13), rather than through the mean point as indicated in the list of formulae. Also failure to work to a sufficient degree of accuracy was a common reason for the loss of marks. Examiners remarked that some candidates failed to conform to the instructions regarding the scaling of axes and that they did not always plot their found regression line accurately. Providing candidates commented on the suitability of using the line to estimate d when v was 55 miles per hour, Examiners gave credit to not only reasoned arguments in favour of using the line, but also to arguments declaring its unsuitable based on the apparently non-linear trend in the data plot.

0.99, Consistent with a linear relationship; $d = 1.64\,v - 25.5$, 65m.

Q16 Most candidates recognised the underlying geometric distribution and part (i) was done well. In part (ii) those who opted for the safety of adding relevant probabilities did not always have the correct number of terms and those who chose the quicker subtraction method often had one of the components incorrect. Part (iii) was quite well done but the final part proved testing since it required knowledge of the mean and variance of geometric distribution as well as the application of the Central Limit Theorem. Though $\overline{X}$ is a discrete variable, a continuity correction (replacing 5 by 5 1/120 and leading to a final answer of 0.012) was not expected and seldom seen. However a number of candidates mistakenly replaced 5 by 5.5. A wide range of marks was obtained on this question. However there were some excellent solutions.

(i) 0.11, (ii) 0.32, (iii) 0.094, (iv) 0.013.

Paper 3

General comments

The question paper proved to be accessible to all apart from the weakest, as practically all of the candidates could make a start on all of the questions. All the evidence indicated that it was a lack of knowledge of the mechanical ideas rather than a lack of time, which prevented candidates from being successful with the more testing parts of the later questions.

Generally the presentation of the solutions was good and even those candidates of modest ability attempted to explain what they were doing, so that method marks could be awarded if the solution started to go wrong through some minor algebraic or arithmetical error.

Candidates could often help their case if they took the trouble to draw a simple diagram with all the given information on it. They then would have avoided, for example in Q.5, opening statements like $6 = \frac{1}{2}gt^2$ or $0 = 15 - gt$. Or again in Q.8 (ii) the direction of the force exerted would have been more obvious and there would have been less error through the interchange of the masses of the trucks.

As ever statics questions seemed to pose the usual problems with the usual uncertainties of the points of application and redirections of forces acting upon a body. In Q.7 and Q.13 it was frequently not appreciated that moments had to be taken, and even when it was realised the taking of moments was badly done.

Comments on Individual Questions

Q.1 This question was generally well answered, the usual failure being faulty removal of brackets in the numerator when the quotient rule was used.

$$\frac{5}{(2x+1)^2}.$$

Q.2 Again candidates scored highly in this question as most appreciated that two applications of the cosine rule would enable them to solve the problem. Any failure was usually due to careless arithmetic, particularly with signs (e.g. cosABC = +1/4)

$$2\sqrt{6}\,(=4.9)$$

Q.3 Nearly all of the candidates made a successful use of the given tree diagram and failure in (i) was a rarity. Finding the value of *p* presented no difficulties to most candidates, but the answer p = 20 appeared frequently, being obtained by equating $1/3(1 + p)$ to 0.35p. The alarming thing was that *p* being greater than unity did not seem to deter candidates from using this value in the final part of the question. Generally, only the abler candidates could cope with this part of the question. Even when it was realised that the word 'conditional' signalled that the answer was going to be obtained from the consideration of a fraction the denominator was often 2/3 rather than 0.65. The numerator 19/30 was usually offered as the final answer

0.05; 38/39 = 0.97

Q.4 The response expected in (a) (i) was either a pie chart or a bar chart and one of these answers was usually given. In (ii) a histogram was the most popular correct answer given, although a stem and leaf diagram or a cumulative frequency diagram were both equally acceptable. The mark for (iii) was rarely given as the response lacked detail, 'Bar Chart' was considered too vague. At the very least examiners were looking for 'Double Bar Chart.'

The next part of the question was well done and most candidates seemed to realise that the interquartile range is a number (11) and not 32 – 43.

Many marks were lost in (b) (ii) because the question asked for a box and whisker plot, but many just gave a free hand sketch. The plot demanded a linear scale from 0 – 60, preferably on graph paper, with the ends of the whiskers at 5 and 60. (The omission of zero failed to illustrate the spread of marks obtained by the candidates compared with the spread of marks available.) With the box in its correctly plotted position, a more meaningful picture of the distribution of the marks would have been obtained.

Median = 32; Interquartile range 11.

Q.5 This question was well answered with the majority opting for the direct method substituting into the formula $s = ut + \frac{1}{2}at^2$. In addition to some of the errors already mentioned made by the weaker candidates, another way was to substitute $a = tg$. Those candidates who adopted the method of first finding the speed of the ball at a height of 6m often failed to find the second time at which the ball was at this height.

0.47 and 2.6 seconds.

Q.6 There was a high degree of success with this question. A few candidates either thought that the retardation was the same over the last ten seconds of the motion, or drew the sketch carelessly and so the shape of the (t, v) graph was a triangle leading to the erroneous distance calculation $s = \frac{1}{2} \times 20 \times 40 = 400$m.

In (ii) the abler candidates gave the correct sketch of a curve with a positive decreasing gradient, but many attempts were either linear graphs or had an increasing positive gradient.

450m

Q.7 The response to this question was poor and full marks were rarely rewarded. It was either omitted altogether or there was an attempt to find F by resolving only. Even those that knew they had to take moments and were also aware that angle BAC = $\tan^{-1}(1/2)$ had difficulty in knowing which length to find in order to find the moment of F about A.

In (ii) the force acting on the lamina at A was often assumed to be vertical or acting along AB. Moments were often taken unnecessarily and incorrectly about B or C. It did not seem to be realised that the force was merely $\sqrt{(F^2 + W^2)}$ and that the angle required was $\tan^{-1}(W/F)$. Even some of the few that got this far correctly still fell into error in that their diagrams indicated all too clearly that they were finding the force of the lamina on the axis rather than the requested force of the axis on the lamina.

$\frac{5}{4}$W, $\frac{W\sqrt{41}}{4}$(=1.6W), 39°.

Q.8 The momentum principle was well understood and only those few who misread the question, and had the trucks approaching each other, failed to find the speed of the trucks after the collision.

In (i) the majority chose the easier approach of applying Newton's Second Law of Motion to the truck A. part (ii) seemed to present more of a problem, as there was confusion with signs. Many candidates either did not seem to know whether they had found a negative acceleration or a positive deceleration, or uncertain which direction the force on the truck B was acting. Most candidates who were successful appreciated that from Newton's Third Law of Motion, the force of B on A was the same and took the easier route of applying the equation of motion to truck A.

$\frac{4}{3}(=1.3)\text{ms}^{-1}$, $\frac{1}{6}(=0.17)\text{ms}^{-2}$, $3333(=3300)\text{N}$.

Q.9 Compared with previous years, there was a welcome improvement with this type of question and the majority of the able candidates appreciated that this was a question concerning the energy principle. Consequently there were many all-correct solutions. A few chose the slightly longer method of first finding the speed (3.43m/s) at the instant he string became taut and then incorporating a P.E. term $0.5g \times (1.2)$ into the energy equation. The most frequent error was to ignore the P.E term or the E.P.E term from the equation. Inevitably the weaker candidates ignored energy application altogether and assumed that the answer came from an application of $v^2 = u^2+2as$. This betrayed a total ignorance of the fact that a body under the action of a varying force cannot have a constant acceleration.

$\frac{1}{4}g(=2.5)\text{N}$, $4.852(=4.8 \text{ or } 4.9)\text{ms}^{-1}$.

Q.10 The first part of the question was well done, but regrettably some did not read the question properly and gave the angle which the strings made to the horizontal. Some of the errors of the weaker candidates were starting with $T\cos\theta = W$. Having different tensions in the two parts of the string and writing the weight of the ring as Wg.

In the next part many candidates got no further than the equations of the type $T_1\cos\alpha + T_2\cos\beta = W$. Even if the tensions were the same they were not aware of relationships of the type $\cos\beta = \sin\alpha$. It was strange that those candidates who persisted with the different tensions did not make the same error in Q.14(a) where a string over a smooth peg is the same set up as a string over a smooth ring. Consequently only the better candidates were able to obtain an equation in terms of the trigonometric rates of one angle only. (E.g. $3\sin\alpha = \cos\alpha$). Having found the correct angles it was surprising how many forgot to complete the question by finding the tension in the string.

41°; 18° and 72°; $\frac{W\sqrt{10}}{4}(= 0.79W)$.

Q.11 This proved to be a popular question and there were many all-correct solutions. Obtaining the formula for the range posed few problems and stating that there was either negligible air resistance or that the acceleration was constant were acceptable as assumptions.

The majority of the candidates then sensibly used the result obtained, both to verify that $u = 121$ and to find D and the required angle of elevation. A number of candidates seemed to want to make a fresh start with equations like $D - 100 = u\cos30°t$ and $D + 100 = u\cos45°t$. Unfortunately these times were usually taken to be the same and so were equated in an attempt to find D.

$D = 1393$ (= 1400)m; 34.45° (= 34° or 35°).

Q.12 Candidates coped easily with the first part of the question which was almost a routine exercise, for many, however this was the only credit gained in the question.

Despite the hint in (i) to use Hooke's Law many embarked on setting up the circular motion equation $T_{OP}\sin\theta + T_{PQ} = M(PQ)\omega^2$ and using this equation alone hoped that somehow an expression for T_{PQ} in terms of m, g and θ would emerge. Sometimes an attempt on the lines hoped for was made later in the question, but often examiners had difficulty in deciding whether the candidate was attempting to answer part (i) or part (ii) of the question.

Some of the errors made by those attempting to find T_{PQ} were (a) to misread the question and find the tension in OP, (b) to have a natural length, l rather than ½l and (c) to have the negative extension $\left(\frac{1}{2}\ell - \ell\sin\theta\right)$.

Only the better candidates could cope with (ii), the most frequent error of the unsuccessful being to omit $T_{OP}\sin\theta$ from the equation of motion. Some of the weaker candidates inappropriately used the result obtained from the different situation in the first part of the question.

$$T_{PQ} = mg(2\sin\theta - 1); \qquad \theta = 45°.$$

Q.13 The requested inequality was usually established by resolving parallel to the plane, but there was often a glossing over when going from $\mu = ¾$ to $\mu \geqslant ¾$. Again most candidates could resolve to obtain the next quoted result although it was obvious, in some cases, that there was some fudging of the signs when the frictional force had obviously been acting initially up the plane.

With the next request success was minimal as very few candidates appreciated that it was necessary to take moments, preferably about the bottom right hand corner of the cube. If moments were taken about any other point credit could only be gained if it was realised that, on the point of toppling, both the frictional force and the normal reaction of the plane on the cube acted through the bottom right hand corner. Some candidates assumed wrongly that the diagonal of the bottom right hand corner was horizontal and the moment of the weight was $W.\frac{1}{2}\ell\sqrt{2}$. A wrong moments equation which appeared frequently even from the very good candidates was $P.\ell = (W\cos\alpha)\frac{1}{2}\ell$. This type of error has occurred with regularity over recent years. If the weight is to be considered as two components $W\sin\alpha$ and $W\cos\alpha$ parallel and perpendicular to the plane respectively, then no credit can be given unless the moment of both components is taken into account in the moments equation.

The final part demanded some calculation to determine how equilibrium was going to be broken but generally all that was given was a general comment on the toppling versus sliding theme with the occasional "hence" to give the impression that some logical argument was being conducted. What was expected was that either the least value for P was calculated for sliding to occur $\left(\frac{6}{5}W\right)$ or the least value of μ was calculated for toppling to occur (1/8).

Toppling occurs first as either $\frac{7}{10}W < \frac{6}{5}W$ or $\frac{1}{8} < \frac{3}{4}$.

Q.14 The first part of the question posed few problems for most candidates. This was after all, the basic type of pulley question and errors from the less able candidates were usually of an arithmetical nature. The acceleration was usually found as part of the process of calculating the tension in the string, and all, except those few who now insisted that the acceleration was now g, knew how to find the time as 4/a. With this type of numerical question it is expected that answers are evaluated to at least 2 significant figures, and not left as a multiple of g.

In (b) the usual approach was to try to form two equations of motion parallel to the plane which contained a frictional force, the tension in the string and the component of the weight. It was not always appreciated that in the motion P moved down the plane and often the direction of the frictional forces on P and Q were equal. What was expected was that the tension would be eliminated from the equation and the resulting equation (or inequality) which could have been found directly by stating $0.3g\sin30° - F_P - F_Q - 0.2g\sin30° > 0$ for the system to move solved for μ. Many candidates equated the forces to zero and there were many less than convincing arguments, which purported to justify the inequality $\mu < \frac{1}{5\sqrt{3}}$ from the equality $\mu = \frac{1}{5\sqrt{3}}$.

If the acceleration terms had been retained after applying Newton's Second Law of Motion the key factor to produce an inequality was to state that when movement occurred the acceleration was greater than zero. A good proportion of candidates made a lot of progress with this problem but invariably came to a halt when they did not know what to do with the acceleration term.

2.354 (= 2.4)N, 2.04 (= 2.0) seconds.

Q.15 Most candidates experienced little difficulty in finding the acceleration of the car. However many of the less able candidates seemed to be confused between the driving force of the engine and the resultant force acting on the car. A frequent error was to substitute into the expression $P = Fv = mav$ to give $a = 1\text{ms}^{-2}$. An alternative, but still incorrect approach was to state that $10000 = (400a - 200)25$.

Finding the time taken at the constant acceleration posed no problems, but for most candidates solutions terminated at this point. One of the objects of this exercise was to emphasise the difference in approach required between a body moving with a constant acceleration and one with a variable acceleration caused by the application of the variable force $\frac{10000}{v}$. Apart from a few good solutions attempts at (iii) were usually of the form $400\frac{dv}{dt} = 200$ or worse $\frac{dv}{dt} = \frac{v-u}{t}$. Some even thought that putting $a = 20$ and $b = 30$ in the given indefinite integrals would solve the problem. Even some of the candidates that wrote out the correct differential equation were let down by faulty algebra when attempting to separate the variables. A similar catalogue of errors appeared in attempts at (iv).

The response to (v) was most disappointing with only a handful of correct solutions. Those who attempted it occasionally picked up a mark for stating that the work done against the resistance was $200s$ but rarely was there an attempt at an equation which contained K.E. terms, $200s$ and the work done by the driving force ($10000t$).

$\frac{1}{2}\text{ms}^{-2}$; 20seconds; 527.3 (= 530) m.

Paper 4

General Comments

In general this was an accessible paper giving the average candidate a chance to attempt every question with some success.

The majority of candidates had sufficient time to complete the paper.

Questions involving written explanations were better attempted than in previous years, but candidates should avoid wasting time by writing too much.

Questions 9,12,14 differentiated well and Questions 13 and 15 were well attempted.

Generally speaking, candidates presented their answers well and examiners had little difficulty in following their solutions.

Comments on Individual Questions

1 Candidates were able to answer this question with reasonable confidence. Most candidates used the quotient rule, though some rearranged and used the product rule or rearranged by dividing. Errors on these more unusual methods included inability to apply the chain rule correctly. On the more straightforward quotient rule method algebraic mistakes including sign errors and errors in removing brackets were very common.

$$\frac{5}{(2x+1)^2}.$$

2 This question involved use of the cosine rule in two different triangles and most candidates who attempted the question were able to apply the rule correctly twice and find AD. Other methods were possible but seldom seen.

4.9.

3 Part (i) and (ii) of this question were well attempted with almost all candidates scoring highly. However, not many candidates were able to correctly find the conditional probability. Some candidates realised that a fraction was required and were able to find the correct form for either the numerator or denominator but many candidates obtained a fully correct fraction.

(ii) 0.05, 0.97

4 (a) Not many candidates found the best diagrams for all the sets of data given, so very few candidates scored 3 marks. Candidates did not appreciate the difference between bar charts and histograms, and very few candidates suggested a dual bar chart for (iii).

(b) Most candidates successfully found the median and interquartile range but unfortunately the box-and-whisker plot was less successfully attempted. Some candidates correctly showed a linear scale from 0 to 60 as required and plotted the diagram with varying degrees of accuracy. Examiners would like to have seen more use of graph paper for the plot; an inaccurate sketch was not acceptable.

(a)(i) pie-chart, (ii) histogram, (iii) double bar chart; (b) 38, 11.

5 Most candidates realised which was the correct sampling procedure. Some candidates considered size of sample but such comments were not relevant here. Many candidates wrote very long answers which were not required.

Early customers may not be typical of customers in general. Useful criticisms could be gained from people who do not use her shop.

6 Many candidates scored well on this question though quite often after successfully finding the two values of p candidates then forgot to find the two possible values of E(X) thus only scoring 5/6. Weaker candidates found the question challenging and some very poor attempts at solving a quadratic equations were seen.

1 and ½.

7 This was generally a well-attempted question with most candidates able to find the correct Binomial forms for part (i). The normal approximation was usually correctly chosen and applied in (ii) though some candidates used a Poisson approximation. Whilst this was not the best approximation to use some credit was given for correctly applying it.

(i) 0.051, (ii) 0.064.

8 Most candidates correctly used the Poisson distribution to find the probabilities required in parts (i) and (ii). The last part was, however, not quite as successfully attempted with many candidates using Po(1) rather than Po(5).

(i) 0.20, (ii) 0.24, (iii) 0.73.

9 Few candidates made a really good attempt at this question. Often candidates did not read the information given correctly. There were many different ways to solve this problem (either using time considerations or cost considerations) and candidates often confused the two. A common error in (ii) for those who used this method was to calculate (500-30)/20 rather than (500-300)/20. Candidates were often able to calculate the mean of the new distribution but calculation of the standard deviation caused problems particularly in (ii).

(i) 0.98, (ii) 0.057.

10 Most candidates were able to find the unbiased estimates in (i) and made a reasonable attempt at the confidence interval in (ii). The last part however was more challenging with sign errors in the z value (-1.28) being commonly seen.

(i) 8900, 503000, (ii) $8740 < \mu < 9060$, 7990.

11 The question required a binomial test to be carrie out and it was surprising how few candidates were able to successfully do this. Many used a Normal approximation and even from those who did attempt a Binomial test many only calculated Pr(2) and compared this to 0.1 rather than Pr(0) + Pr(1) + Pr(2). Candidates were able to recover on the second part of the question where a normal approximation was required. A common error here, however, was to use B(40, 0.3) rather than B(40, 0.2).

Accept the null hypothesis, Reject the null hypothesis.

12 This question proved demanding with some candidates omitting it completely. In (i) very few candidates realised that the sample should be taken from a normal population; candidates incorrectly suggested that the sample should be normal. Many candidates confused biased and unbiased estimates for the variance and hence lost marks, and many did not justify their

conclusions by showing clearly (using a diagram or an inequality statement) the comparison between the t value calculated and the table value. Few candidates scored fully on the last part.

(i) Data must be a sample from a normal population, (ii) 31.

13 This question was well attempted with candidates able to calculate the product moment correlation coefficient correctly. The interpretation was not quite so well done. The equation of the regression line was also well attempted though some candidates after correctly finding the gradient of the line then went on to use a data point to find the equation. Candidates were able to plot the data and their line successfully though some candidates did not use the scales given. Several different sensible suggestions as to the suitability of the regression line were accepted. Some candidates suggested it was not suitable as the data points formed a curve and others suggested that it was suitable as the value of r was very high. These were both acceptable comments. In general candidates scored well on this question.

0.99, Strong positive correlation, $d = -25.5 + 1.64v$, 0.65,
eg Plot of data shows evidence of a non-linear relationship.

14 This was a good question which differentiated well. Part (i) and (ii) were usually correctly done but in (iii) not many candidates realised they needed to find the two cases FSS, SFS and add these probabilities together. It was disappointing in the last part that many candidates did not know that the mean of a Geometric distribution was $1/p$ and that q/p^2 was the variance. All too often candidates incorrectly used np and npq. Many candidates omitted to use the standard error and some used a continuity correction which was not acceptable.

(i) 0.11, (ii) 0.32, (iii) 0.094, 0.013.

15 Many candidates scored well on this last question and it was pleasing to note that candidates in general did not round their answers at an early stage but kept a good level of accuracy throughout. Some candidates gave contradictory conclusions having not set up a correct null and alternative hypothesis. In (ii) some candidates had difficulty in obtaining 207/400 for the Poisson mean. Some calculated boys and girls separately. The most common error was to calculate the expected frequency of exactly 3 rather than 3 or more. Again in (ii) candidates should have set up a null and alternative hypothesis to test.

(i) There is no association
(ii) Poisson distribution is not a good fit.

(b) Most candidates applied the difference method accurately to obtain the sum function S_N for $\sum_{n=1}^{N} u_n$.

(i) As in Q.7, not all candidates comprehended the meaning of a positive quantity raised to a zero power so that the result $S_N = -1 - \cos\left[\pi\sqrt{(N+1)}\right]$ for this special case, did not always appear. Nevertheless, most concluded that the infinite series, S does not converge, though a persistent error was the supposition that the terms 'non-convergent' and 'divergent' have identical meanings.

(ii) The result $S_N = -1 - \cos\frac{\left[\pi\sqrt{(N+1)}\right]}{N+1}$ for this case appeared in most responses. Beyond that, candidates were expected to say that S is convergent and also that $S_\infty = -1$, even though it is obvious that the second of these two statements implies the first.

(b) $S_N = -1 - \cos\frac{\left[\pi\sqrt{(N+1)}\right]}{(N+1)^a}$; (i) S does not converge; (ii) S converges and $S_\infty = -1$.

Q.13 Although most candidates had an understanding of the basic ideas of this question, few worked accurately in all parts of this question and moreover there were many notational errors.

(i) Very few candidates failed to obtain the modulus of the given complex number. The argument was usually written down correctly, though it was not always given in the stipulated range.

It was generally perceived that for each of the 4 complex numbers $z_k (k = 1, 2, 3, 4)$ which are roots of the given equation, mod(z_k) = 2. In contrast, many errors appeared in the formulation and derivation of the arguments. In some cases 'i' was missing altogether in the form $re^{i\theta}$.

(ii) Most candidates understood that the equation given in this part of the question has 8 roots, of which 4 are the fourth roots of -1, and the remainder are the complex numbers obtained in (i). However, about a third of all responses showed incorrect solutions for the equation $z^4 + 1 = 0$, and in any case, errors in part (ii) precluded any possibility of a complete and correct response to this part of the question.

(iii) The majority of diagrams in responses were complete and correct and also well drawn. Arguments to determine how many of the 8 complex numbers found in part (ii) are represented by points in R were often clear enough, though on account of earlier errors there were many incorrect conclusions.

(i) 16, $\frac{-\pi}{3}$: $2e^{iq\pi}$, where $q = \frac{-7}{12}, \frac{-1}{12}, \frac{5}{12}, \frac{11}{12}$; (ii) $e^{is\pi}$, where $s = \frac{-3}{4}, \frac{-1}{4}, \frac{1}{4}, \frac{3}{4}$ and $2e^{iq\pi}$, where $q = \frac{-7}{12}, \frac{-1}{12}, \frac{5}{12}, \frac{11}{12}$; (iii) 2.

Q.14 This question, which is the most extensive of the paper, generated a lot of good work, and a few candidates obtained full marks.

(i) Most responses here could be said to be complete, though very often they wandered out into situations of unnecessary complexity. However a significant minority of candidates produced arguments of the form '$ab = ba^3 \Rightarrow \ldots \Rightarrow b = b$', but made no attempt to show that the direction of their argument was reversible.

(ii) Only a minority of responses showed a complete and correct listing of the orders of the elements of G. It was common for 3 of the elements to be assigned the order 2.

(iii) It was expected that almost all candidates would find this part of Q.14 extremely easy, but this proved not to be the case. About a quarter of all responses stated that there are 3 subgroups of G of order 2 and there were other incorrect entries in the given table.

(iv) Most candidates were able to identify 2 subgroups of order 4, but relatively few were able to find the remaining subgroup of this order. Moreover some subsets which are not subgroups of G appeared.

Only a minority were able to produce a complete and correct argument with which to conclude both this question and the paper. For arguments based on orders of elements it is essential that sufficient specific results are shown to justify conclusions. In any case, it is easy to see without reference to orders of elements, that K is not isomorphic to either G or H, for G and H are non-commutative whereas K is commutative. For the $G - H$ comparison, it is sufficient to observe that in G there is only 1 element of order 2, namely a^2, whereas in H, as c^2 and d are both of order 2, then H has at least 2 elements of order 2 and so cannot be isomorphic to G.

(ii)

Element of G	e	a	a	a	b	ab	a^2b	a^3b
Order	1	4	2	4	4	4	4	4

(iii)

n	2	3	4	5	6	7
Number of subgroups of order n	1	0	3	0	0	0

(iv) $\{e, a, a^2, a^3\}$, $\{e, a^2, b, a^2b\}$, $\{e, a^2, ab, a^3b\}$.
Arguments and conclusions for the final part of this question are given above.

MATHEMATICS 9200

Component threshold marks

Component	Maximum Mark	A	B	C	D	E	N	U
1	120	82	71	61	51	42	33	0
2	120	89	79	68	58	48	38	0
3	120	85	75	66	57	49	41	0
4	120	91	82	72	62	53	44	0

Overall Threshold Marks

Option	Maximum Mark	A	B	C	D	E	N	U
A (1,2)	240	171	149	129	109	90	71	0
B (1,3)	240	164	144	125	107	89	71	0
C (1,4)	240	171	150	131	113	95	77	0

The percentage of candidates awarded each grade was as follows

Grade	A	B	C	D	E	N	U
%	36.8	16.1	13.3	12.7	8.6	7.1	5.3
Cumulative %	36.8	52.9	66.3	79.0	87.6	94.7	100.0

The total candidature was 1395